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Generalized flux vacua

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Abstract: We consider type II string theory compactified on a symmetric $T^6/\mathbb{Z}_2$ orientifold. We study a general class of discrete deformations of the resulting four-dimensional supergravity theory, including gaugings arising from geometric and “nongeometric” fluxes, as well as the usual R-R and NS-NS fluxes. Solving the equations of motion associated with the resulting $\mathcal{N} = 1$ superpotential, we find infinite families of supersymmetric vacua with all moduli stabilized. These solutions have parametrically small string coupling and moduli masses, although we expect that a complete string realization of these models would suffer from large α' corrections. We also describe some aspects of the distribution of generic solutions to the SUSY equations of motion for this model, and note in particular the existence of an apparently infinite number of solutions in a finite range of the parameter space of the four-dimensional effective theory.

Keywords: String Duality, Superstring Vacua, Flux compactifications.

Contents

1. Introduction

Classifying the full space of string theory vacua is an enormous problem. For any given string theory, there is a broad range of possible compactification geometries. Imposing supersymmetry simplifies the problem somewhat, and for many years attention was focused on Calabi-Yau manifolds as the primary structure needed for supersymmetric compactification of string theory. In recent years, however, more and more attention has focused on compactifications including fluxes of various p-form fields on topologically nontrivial cycles of the compactification manifold [1 − 3]. This has led to the realization that a broader class of manifolds are needed even for supersymmetric compactifications [4, 5]. It is possible, however, that concrete realizations of compactifications explored to date only scratch the surface of the full range of possibilities. In general, supersymmetric compactification on a Calabi-Yau or other related manifold with fluxes leads to a gauged supergravity theory in four dimensions. In many gauged supergravity theories, the parameters associated with the gauging of the supergravity can be identified in this way with fluxes on a compactification manifold or with certain features of the geometry of the compactification manifold [6 − 8].

In other cases, however, it is not known how to interpret the gauging parameters in ten dimensions. It is possible that in some cases such gaugings are the result of compactifications on nongeometric string backgrounds [9, 10]. Thus, while many interesting four-dimensional solutions, including stable de Sitter vacua [11], may be realized in effective four-dimensional supergravity theories, we often do not know how to interpret these solutions in terms of ten-dimensional string theory.

When two ten-dimensional theories are related through a duality symmetry, such as T-duality or mirror symmetry, we have a situation where a given four-dimensional effective theory may be derived in two different ways. In this case, gauging parameters which have a natural interpretation as fluxes in one picture may not have a known interpretation in the dual picture. The existence of one picture in which these gauging parameters can be understood in ten dimensions, however, suggests that these parameters should be allowed to take nonzero values in a more general context, and that it may be possible to turn on arbitrary combinations of these parameters even when there is no single picture in which they all have a known interpretation. This observation has motivated analysis of the duality transformation properties of the gauging parameters in four-dimensional supergravity theories [12]. In [9], we considered a simple example of this situation, namely, a toroidal orientifold in Type II string theory. We derived the four-dimensional superpotential for this toroidal orientifold, including NS-NS and R-R fluxes in both the IIA and IIB theories. These fluxes give rise to different subsets of the set of possible gaugings of the four-dimensional model. We argued that to have a class of models which are invariant under duality, it was natural to include a full duality-invariant set of gauging parameters. When we consider T-duality transformations on the toroidal orientifold, the duality-invariant set of gauging parameters includes integers which appear as T-duals of NS-NS 3-form flux H. These integers can be interpreted as parameterizing twists on the torus, often called "geometric flux" (as can be found after one T-duality using the Buscher rules [13]) and as generalized fluxes (after two or three T-dualities).[1]

These generalized fluxes can be referred to as "nongeometric," in the sense that when they are present in the absence of other fluxes, they describe backgrounds with no global (or possibly even local) geometric interpretation. There is so far no complete worldsheet description of spaces with these fluxes as string backgrounds, but one proposal for a stringy construction of these fluxes has been described by Hull [14] and explored by several authors at both the classical [15 – 17] and quantum [18] levels. While simple combinations of geometric and generalized fluxes may have a string interpretation along these lines, when sufficiently complicated combinations of fluxes are included, we have as yet no concrete idea of how to lift the resulting four-dimensional physics to a full string compactification. While we touch on this issue here, for the most part we simply treat the fluxes as parameters in the four-dimensional effective theory. Other approaches to the worldsheet description of nongeometric backgrounds include asymmetric orbifolds [19, 20], and a recent generalization [21]. We do not expect, however, that the backgrounds described in the present work

[1]Here the word "generalized" is not used in the sense of "generalized geometry" [5]. We use it instead to describe essentially topological structures which are related to p-form flux under duality.

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generically have an asymmetric orbifold description.

In this paper we consider the four-dimensional theory with this general set of fluxes and solve the tree-level supergravity equations. We find classes of vacua in which the string coupling g and cosmological constant Λ are parametrically controlled. These vacua are reminiscent of the parametrically controlled IIA vacua described in [22], although in that case there was an explicit picture of the vacua as ten-dimensional compactifications on a large radius Calabi-Yau manifold. Here, we do not have a ten-dimensional picture of how these vacua should be interpreted, and likely have no control over α' corrections. Indeed, the structure of the fluxes suggests that to construct a complete lift of these solutions to a string compactification may require the introduction of additional light fields. Because the fluxes capture some essentially topological features of the theory, however, we believe that the existence of these SUSY vacua points to some new class of either geometric or nongeometric string vacua which we may understand more clearly in due course. While the detailed properties of true string theory solutions related to the four-dimensional vacua we exhibit here are likely not to be robust under the inclusion of further stringy effects, broad features such as (for example) the existence of an infinite numbers of solutions may persist. Furthermore, we find that if we neglect α' corrections it is possible to control those parameters which are accessible from our knowledge of the 4D supergravity theory alone — namely the value of the string coupling constant, the cosmological constant, and the masses of the fields retained in the 4d theory. Solutions where these quantities are controllably small are promising starting points for a future string theoretic realization of these vacua.

In addition to constructing infinite families of vacua for the 4D effective theory, we will also study the broad features of the general supersymmetric solutions in this model. The generic solutions stabilize all moduli at tree level, and are not equivalent to known purely geometric compactifications in any duality frame. We find that our toroidal model possesses an apparently infinite number of (tree-level) solutions in finite ranges of parameter space. This somewhat surprising result seems to contradict recent predictions regarding properties of the string landscape [23, 24], though as we will discuss there are some reasons why the solutions we find may not correspond to stable nonperturbative vacua in a complete string theory framework.

In section 2 we review the construction of the superpotential for the dimensionally reduced four-dimensional theory on the toroidal orientifold, including generalized fluxes. We give a suggestive new concise description of the constraints on consistent generalized fluxes in terms of a generalized derivative operator. In section 3 we give the equations of motion and briefly discuss some simple classes of solutions. In section 4 we describe two two-parameter families of SUSY vacua, and in section 5 we briefly describe the statistics of more general vacua. section 6 contains some concluding remarks.

2. Review and preview

2.1 Generalized fluxes

In [9] we identified a set of NS-NS "fluxes" on the torus, which can individually be obtained

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from the usual three-form H-flux by T-duality. T-duality acts on the integrated NS-NS fluxes according to the chain

$$\bar{H}_{abc} \overset{T_a}{\longleftrightarrow} f^a_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}_c \overset{T_c}{\longleftrightarrow} R^{abc}. \tag{2.1}$$

Here $\bar{H}_{abc}$ is the integer number of units of H-flux on the (abc)-cycle of the torus. The "geometric flux" f^a_{bc} characterizes the field strengths of a basis of one-forms η^a satisfying

$$d\eta^a = f^a_{bc}\,\eta^b \wedge \eta^c. \tag{2.2}$$

Differentiating (2.2) yields a Jacobi identity, and thus the f^a_{bc} may be viewed as the structure constants for a Lie algebra. The one-forms η^a can be related to both the generators of the isometry group of the compactification manifold and to a basis for its frame bundle. When the Lie algebra defined by the f^a_{bc} is nilpotent, the η^a can be constructed explicitly, and the geometric flux can be straightforwardly interpreted as adding twists to an underlying torus. When the algebra is not nilpotent, then the compact space is less closely related to the torus, and from the four-dimensional point of view the lift to ten dimensions is more subtle [7]; we will return to this point below.

Following the T-duality chain of equation (2.1), acting on f^a_{bc} with an additional T-duality in the direction b takes us to the nongeometric structure characterized by Q^{ab}_c.[2] For a torus with N units of such a Q-flux, and no other NS-NS fluxes, fields are periodic only up to an identification which mixes geometric data with the (integrated) B-field, and R-R p-forms of differing degree with each other. While the resulting compactification has a geometric description locally, the transition functions render the global description nongeometric.

Finally, the last step in the T-duality chain is a formal T-duality on c, taking us to an object R^{abc}. This is the NS-NS analogue of the R-R T-duality rule which takes $\bar{F}_x \overset{T_x}{\longleftrightarrow} F_0$. A compactification with R-flux has apparently no geometric description, even locally. A heuristic argument for this lack of local geometry goes as follows: Consider a T^3 with nonzero H flux. It is not possible to wrap a D3-brane on this T^3 for the simple reason that turning on H-flux would give a nonzero Bianchi identity for the gauge field living on the D3-brane, $dF_2 = H_3 \neq 0$. T-dualizing this configuration once would give a D2-brane on a twisted torus. However, the 2-cycle the D2-brane should wrap no longer exists in the integer homology of the twisted torus. Thus, like the original D3-brane, this D2-brane can not exist. If these branes cannot exist, neither can the D0-brane one would get by performing two more T-dualities. This indicates an apparent lack of the notion of spacetime points on a T^3 with R-flux. It is in this sense that we say that the R-flux gives a background which is not even locally geometric. A further discussion of the properties of branes in backgrounds with nongeometric fluxes appears in [17].

We need some understanding of how to combine these generalized flux objects in a consistent manner. These new structures introduce new terms in the NS-NS Bianchi identities

[2] Strictly speaking, the isometry in the direction b is not globally defined, but it seems that the T-duality in this direction can be understood, at least in the dimensionally reduced theory, in terms of an orbifold by a combination of a shift and a T-duality symmetry; see also [16].

and the R-R tadpoles/Bianchi identities. The need to satisfy Bianchi identities and tadpole cancellation conditions constrains the admissible fluxes in a general compactification. For the R-R fluxes, the constraints found in [9] can be written concisely as

$$(\bar{H} \wedge + f \cdot + Q \cdot + R\llcorner)\bar{\mathcal{F}} = 0. \tag{2.3}$$

Here $\bar{\mathcal{F}}$ is the formal sum of the integrated R-R field strengths, and we have used the notation

$$R\llcorner\omega^{(p)} \equiv R^{abc}\omega_{abca_4\ldots a_p} \tag{2.4}$$

in addition to

$$f \cdot \omega^{(p)} = f^{a}_{[bc}\omega_{|a|a_2\ldots a_p]}, \qquad Q \cdot \omega^{(p)} = Q^{ab}_{[c}\omega_{|ab|a_3\ldots a_p]}, \tag{2.5}$$

contracting all upper indices and antisymmetrizing all uncontracted lower indices. This is the natural action of a mixed tensor on a p-form. Recalling that the standard R-R tadpoles/Bianchi identities can be written in terms of an H-twisted differential operator,

$$(d + H\wedge)\mathcal{F} \equiv d_H\mathcal{F} = 0, \tag{2.6}$$

the constraints (2.3) have a suggestive interpretation as the action of an operator defined by acting with all generalized NS-NS fluxes,[3]

$$\mathcal{D} \equiv H \wedge + f \cdot + Q \cdot + R\llcorner \tag{2.7}$$

on the R-R fluxes $\bar{\mathcal{F}}$,

$$\mathcal{D}\bar{\mathcal{F}} = 0. \tag{2.8}$$

Moreover, just as the usual Bianchi identity for H,

$$dH = 0, \tag{2.9}$$

can be understood as the condition that the twisted differential $d_H \equiv d + H\wedge$ be nilpotent, the entire set of NS-NS constraints, including the contributions from all generalized fluxes, are equivalent to a nilpotency condition

$$\mathcal{D}^2 = (H \wedge + f \cdot + Q \cdot + R\llcorner)^2 = 0. \tag{2.10}$$

In addition to the (bilinear) generalized Bianchi identities for the NS-NS fluxes, (2.10) also incorporates the (linear) "tracelessness" conditions $f^{a}_{ab} = 0 = Q^{ab}_{a}$.

Now that we have rewritten the constraints in a more covariant form as (2.3), (2.10), we have eliminated the explicit dependence on a particular choice of coordinate system.

[3]In (2.3), there is no term corresponding to the derivative operator d; this is because we have integrated the differential expression to obtain a topological formula. The contraction with f contains the information coming from d when the forms are expressed in terms of the invariant basis η^a on, for example, a twisted torus. We expect that some generalization of these formulas should hold without integration, in which case there should be additional local contributions from d, though it is not clear what this should mean when the compactification space is not a manifold.

This provides a natural point of departure for considering these nongeometric structures on manifolds more complicated than the torus, though we do not pursue this further here.

While any individual nongeometric flux is T-dual to H-flux and has a reasonably straightforward interpretation as discussed above, a general combination of geometric and nongeometric fluxes can never be brought to a description in terms of conventional fluxes in any duality frame, and may be more difficult to interpret. Furthermore, when multiple fluxes are combined, the nature of the compactification manifold may change dramatically. For example, including "geometric fluxes" f^a_{bc} which give rise to an algebra which is not nilpotent gives rise to a space in which the fluxes no longer have the same interpretation as geometric twists. A simple example of this is compactification on S^3, which produces f's which are the structure constants of SU(2) [7]. Performing a T-duality on the fiber of the Hopf fibration of S^3 effectively performs a T-duality on one of the indices of the f's and gives a space which can be described geometrically as $S^1 \times S^2$ with one unit of H flux [25]. By formally raising and lowering the indices on the f's, however, we see that this compactification should be described by fluxes which we would label as one H and two Q's.[4] While this background is not a solution of the equations of motion and is not supersymmetric, it suggests that when the algebra is not nilpotent, the question of lifting the background to ten dimensions becomes more subtle, and that we should be cautious in our assumptions about which backgrounds are geometric and which are not.

To be clear, in this paper we will consider a four-dimensional effective theory with a specific form for the superpotential. Each individual term in the superpotential has a clear interpretation in terms of some individual (generalized) flux on an underlying torus, in the absence of other fluxes. However, for the explicit solutions we find, the gauge algebra of the resulting theory is not nilpotent, and thus the lift of these solutions to ten dimensions is not straightforward. While the individual fluxes appearing in our superpotential may have nongeometric interpretations, we cannot rule out the possibility that there may be a completely geometric description of these vacua in ten dimensions. For simplicity, we will continue to use the language of flux compactification on a torus, but the reader should bear these caveats in mind.

2.2 A simple model: the symmetric $T^6/\mathbb{Z}_2$ orientifold

In this paper we will carry out the study of generalized flux compactifications on the simple symmetric torus orientifold $(T^2)^3/\mathbb{Z}_2$ which we initiated in [9]. As in [9], the underlying compactification we consider is type II on a T^6 orientifold where the complex structure is restricted to be diagonal and symmetric by choosing fluxes which are invariant under cyclic permutations of the three 2-tori. Thus, the complex structure parameters are proportional to the identity matrix, $\tau_{ij} = \tau\delta_{ij}$. We will consider this orientifold in two T-dual descriptions, namely as a IIB compactification with O3-planes and as a IIA compactification with O6-planes. This model has three complex moduli: τ, which is the complex structure parameter of the torus in IIB and the Kähler modulus in IIA; S, the axiodilaton; and U, which is the Kähler modulus in IIB and the complex structure modulus in IIA.

[4] We would like to thank David Marks for pointing out this example to us.

Allowing all generalized fluxes which are consistent with the symmetric restriction yields a polynomial superpotential for the four-dimensional theory:

$$\begin{aligned} W &= a_0 - 3a_1\tau + 3a_2\tau^2 - a_3\tau^3 \\ &\quad +S(-b_0 + 3b_1\tau - 3b_2\tau^2 + b_3\tau^3) \\ &\quad +3U(c_0 + (2c_1 - \tilde{c}_1)\tau - (2c_2 - \tilde{c}_2)\tau^2 - c_3\tau^3) \\ &\equiv P_1(\tau) + SP_2(\tau) + UP_3(\tau). \end{aligned} \tag{2.11}$$

The integer coefficients a_i, b_i, c_i correspond to the number of units of integrated fluxes on cycles of the torus.[5] The a_i come from R-R fluxes in both IIA and IIB. The b_i in IIB come from H-flux on various cycles of the torus, while in IIA they are given by (in ascending order) H, f, Q, R. The c_i in IIB are all due to Q-flux, and in IIA are again given by (in ascending order) H, f, Q, R. For further details, we refer the reader to [9]. In our conventions, the tree-level Kähler potential is given by

$$K = -3\ln(-i(\tau - \bar{\tau})) - \ln(-i(S - \bar{S})) - 3\ln(-i(U - \bar{U}))\,. \tag{2.12}$$

Equations (2.11) and (2.12) determine the scalar potential, which has the usual form

$$V = e^K \left(\sum_{i,j \in \{\tau, U, S\}} K^{ij} D_i W \overline{D_j W} - 3|W|^2 \right). \tag{2.13}$$

The R-R tadpole constraints (2.3) applied to this model yield the restrictions on the integer coefficients

$$a_0 b_3 - 3a_1 b_2 + 3a_2 b_1 - a_3 b_0 = 16 \tag{2.14}$$

and

$$a_0 c_3 + a_1(2c_2 - \tilde{c}_2) - a_2(2c_1 - \tilde{c}_1) - a_3 c_0 = 0. \tag{2.15}$$

The nonvanishing right hand side of (2.14) is the contribution from the orientifold planes. The NS-NS Bianchi identities and tadpoles, in this model, give a set of constraints relating the b_i and the c_i:

$$c_0 b_2 - \tilde{c}_1 b_1 + c_1 b_1 - c_2 b_0 = 0 \tag{2.16}$$

$$c_1 b_3 - c_2 b_2 + \tilde{c}_2 b_2 - c_3 b_1 = 0 \tag{2.17}$$

$$c_0 b_3 - \tilde{c}_1 b_2 + c_1 b_2 - c_2 b_1 = 0 \tag{2.18}$$

$$c_1 b_2 - c_2 b_1 + \tilde{c}_2 b_1 - c_3 b_0 = 0, \tag{2.19}$$

as well as a set of constraints among the c_i:

$$c_0 \tilde{c}_2 - c_1^2 + \tilde{c}_1 c_1 - c_2 c_0 = 0 \tag{2.20}$$

$$c_3 \tilde{c}_1 - c_2^2 + \tilde{c}_2 c_2 - c_1 c_3 = 0 \tag{2.21}$$

$$c_3 c_0 - c_2 c_1 = 0. \tag{2.22}$$

[5]The flux integer c_1 is in the notation of [9] $c_1 \equiv \hat{c}_1 = \check{c}_1$, and likewise for c_2. We will use the notation c_i to refer to all coefficients in $P_3(\tau)$.

Of the seven NS-NS constraints (2.16)–(2.22), four are independent. Including the R-R constraints (2.14)–(2.15) then gives six independent constraints, reducing the number of independent flux integers from fourteen to eight.

Finally, in order to avoid additional subtleties, we impose the further restriction that all flux coefficients should be even. This restriction is motivated by the observation that even numbers of flux quanta on an orientifolded manifold lift to integer quanta on the original manifold, while odd numbers of flux quanta require a more sophisticated construction incorporating exotic orientifold planes [26]. For simplicity, we will thus consider only even numbers of flux quanta for all of the generalized fluxes we consider here. For any individual nongeometric NS-NS flux, this restriction to even integers only can be understood as the T-dual description of requiring the number of units of H-flux to be even. While this argument from T-duality is insufficient to fully describe a general combination of NS-NS generalized fluxes, where the new fluxes can never be T-dualized completely to H-flux, it is a strong motivation to consider only even flux coefficients.

3. Equations of motion

Let us now discuss solutions to the tree level equations of motion in the effective theory defined by the superpotential (2.11) and the Kähler potential (2.12). The scalar potential comes entirely from F-terms, as in this model the remaining fields are not charged under the gauge group, and there is therefore no D-term contribution. Thus, to find supersymmetric solutions, it suffices to solve the F-flat conditions $D_\alpha W = \partial_\alpha W + (\partial_\alpha K)W = 0$.

In Subsection 3.1 we summarize the equations we are interested in solving. In 3.2 we describe some simple special cases of solutions. In 3.3 we discuss the criteria for validity of solutions.

3.1 Equations

The equations of motion we are interested in solving are the F-flat conditions

$$P_1(\tau) + \bar{S}P_2(\tau) + UP_3(\tau) = 0 \tag{3.1}$$

$$P_1(\tau) + SP_2(\tau) + \left(\frac{2}{3}U + \frac{1}{3}\bar{U}\right) P_3(\tau) = 0 \tag{3.2}$$

$$(\tau - \bar{\tau})\partial_\tau W - 3W = 0\,. \tag{3.3}$$

Using equations (3.1) and (3.2), one finds that physical solutions must satisfy

$$\mathrm{Im}\,(P_2\bar{P}_3) = 0. \tag{3.4}$$

One may readily solve for U and S as functions of τ, leaving (3.4) and a twelfth-order (real) polynomial equation for the real and imaginary parts of τ coming from (3.3). Thus, we must simultaneously solve this twelfth-order polynomial and a (real) fifth-order polynomial from (3.4), which in general must be done numerically. Once we have a solution τ_0, S_0, U_0, we can read off the string coupling,

$$g = \frac{1}{\mathrm{Im}\,S_0}, \tag{3.5}$$

and the cosmological constant

$$\Lambda = V(\tau_0, S_0, U_0) = -3e^{K(\tau_0,S_0,U_0)}|W(\tau_0, S_0, U_0)|^2. \tag{3.6}$$

We consider physical solutions to equations (3.1)–(3.3) to be only those with positive values for the imaginary parts of all moduli. In the absence of fluxes, this requirement is straightforward to interpret, as the imaginary parts of the moduli control the string coupling and geometric information about the internal manifold, and must be strictly greater than zero. We assume that this positivity requirement continues to hold in the presence of arbitrary combinations of generalized fluxes, as turning on fluxes simply turns on new couplings in the scalar sector of the four-dimensional theory.

Relatedly, the real (axionic) parts of the moduli are in our conventions periodic with unit period. This periodicity is easily derived in the underlying toroidal compactification in the language of either IIA or IIB. The addition of fluxes, conventional and nongeometric, again only adds couplings between the scalar fields and does not affect the range in which they take values.

Our interest here is to systematically study the solutions to the equations of motion (3.1)–(3.3) for general choices of the integer coefficients a_i, b_i, c_i in (2.11). Not all choices of flux integers admit a physical supersymmetric solution. The choices of flux integers that do yield physical solutions generically fix all moduli and are generically not gauge equivalent to any vacua which have a conventional geometric interpretation in either type IIA or IIB. We will discuss two particularly well-behaved families of such vacua in section 4, and describe the general solutions in section 5.

In order to understand the solution space of this model, we must understand how to avoid multiply counting solutions which are related by the modular group of the compactification. In our simple torus model, the non-compact portion of the modular group is given by the integral shift symmetries of the three axionic scalar fields. The shift $\tau \to \tau + n$ combines with the inversion $\tau \to -1/\tau$ to give a factor of $SL(2, \mathbb{Z})$. The S-duality transformation $S \to -1/S$, which would combine with the shifts $S \to S + m$ to give another factor of $SL(2, \mathbb{Z})$, maps generalized NS-NS fluxes to additional nongeometric structures which mix the metric with R-R fields [9, 27]. As S-duality therefore takes us out of the class of compactifications we consider here, it is not a factor in the modular group and we will not study it further here. The same is true about the inversion transformation $U \to -1/U$. Each of these transformations of the moduli is accompanied by a corresponding transformation of the fluxes to leave the equations of motion invariant. We give the explicit form of all of these transformations in appendix A. These transformations descend from large gauge transformations, and therefore configurations related by these transformations are physically equivalent. In addition, there is a $\mathbb{Z}_2$ factor in the modular group which flips the signs of all fluxes, taking $W \to -W$; this transformation does not produce a physically distinct vacuum and as such is also a gauge transformation.

In addition to the modular group discussed above, there are a number of additional transformations relating vacua which, despite having identical spectra, are physically inequivalent. First, there is a sign flip taking $\tau, S, U \to -\bar{\tau}, -\bar{S}, -\bar{U}$, with an accompanying transformation on the fluxes leaving the equations of motion invariant. Additionally, one may shift the axions by rational numbers, e.g. $U \to U + 1/n$ or $S \to S + 1/n$, which is possible whenever the resulting fluxes are still even integers. Both the sign flip and the fractional shift transformations relate vacua with physically distinct expectation values for the axions but identical spectra of particle masses, coupling constant, and cosmological constant. Thus, we have relations between different four-dimensional theories since the transformations act on the couplings as well as the fields. One can therefore think of theories related by these transformations as being part of a discrete moduli space of a larger theory.

3.2 Some special cases

There are several classes of possible flux configurations which are of special interest: fluxes which have an interpretation as a geometric IIB compactification, fluxes which have an interpretation as a geometric IIA compactification, and fluxes yielding solutions which have a vanishing expectation value for the superpotential, giving a Minkowski vacuum at tree level. Each of these subclasses of flux integers seems to be a set of measure zero with respect to the full set of possible flux configurations admitting physical solutions. We will discuss these cases in this section, and treat more general cases in the remainder of the paper.

3.2.1 Geometric IIB vacua

If one takes all the c_i to vanish, then the resulting compactification is simply a standard IIB flux compactification on the torus, and the superpotential (2.11) reduces to the well-known Gukov-Vafa-Witten superpotential [2]. The resulting vacua on the torus have been studied in [28 – 30]. As is well-known, these fluxes are not sufficient to stabilize all moduli at tree level. While in principle these configurations would appear as a subset of the compactifications we consider here, our main interest in this paper is in vacua with all moduli stabilized, so we will not discuss geometric IIB vacua further here. In particular, these vacua will not be included in the sets of vacua studied in section 5.

3.2.2 Geometric IIA vacua

If the fluxes b_2, b_3 and $c_2, \tilde{c}_2, c_3$ all vanish, then all the remaining terms in the superpotential have a geometric interpretation in IIA; such toroidal compactifications with geometric flux have been studied in [15, 31 – 34]. On the symmetric torus, however, there is an additional interplay between the equations of motion and the constraints which drastically limits the kinds of supersymmetric vacua one may obtain in this way.

Since for a geometric solution in IIA, P_2 and P_3 are linear, equation (3.4) for τ reduces to

$$(\mathrm{Im}\,\tau)(3b_1c_0 + b_0(2c_1 - \tilde{c}_1)) = 0. \tag{3.7}$$

Requiring $\mathrm{Im}\,\tau > 0$ then yields a constraint on the fluxes

$$3b_1c_0 + b_0(2c_1 - \tilde{c}_1) = 0. \tag{3.8}$$

Consider, however, the tadpole constraints for this set of fluxes:

$$3a_2b_1 - a_3b_0 = 16 \tag{3.9}$$
$$a_2(2c_1 - \tilde{c}_1) + a_3c_0 = 0 \tag{3.10}$$
$$(c_1 - \tilde{c}_1)b_1 = 0 \tag{3.11}$$
$$c_1(c_1 - \tilde{c}_1) = 0. \tag{3.12}$$

Let us suppose first that neither b_1 nor $(2c_1 - \tilde{c}_1)$ is zero. Then using (3.8), we can trade the c_i in (3.10) for b_i to find

$$3a_2b_1 - a_3b_0 = 0, \tag{3.13}$$

which is, of course, inconsistent with (3.9).

The only way to avoid this inconsistency is to take either b_1 or $(2c_1 - \tilde{c}_1)$ to be 0. Then the constraints enforce $c_1 = \tilde{c}_1 = c_0 = 0$, for both $b_1 = 0$ and $b_1 \neq 0$. In either case, this is simply an alternate description of a conventional IIB compactification.

If one chooses to cancel the tadpole from the orientifold planes using D-branes instead of fluxes, then one may set the right hand side of (3.9) to zero. The constraints then allow for a wider range of possible solutions, including those with no geometric interpretation in IIB. We will limit our analysis here, however, to the tadpole as given in (3.9), without D-branes.

3.2.3 Vacua with $W = 0$

From the equations of motion, one may show that vacua where the superpotential vanishes at the minimum are obtained only when

$$P_1(\tau_0) = P_2(\tau_0) = P_3(\tau_0) = 0, \tag{3.14}$$

where τ_0 is the value of τ which solves the F-flat equations. It is then clear, as we need $\mathrm{Im}\,\tau > 0$, that a necessary condition for a $W = 0$ solution is that the polynomials P_i all share a common quadratic factor, P, which vanishes at τ_0. The remaining nontrivial equations of motion then fix two of the four remaining degrees of freedom. While Minkowski vacua are very interesting, again, we are focusing our attention on vacua with all moduli stabilized, and therefore will not examine $W = 0$ solutions further. For a novel approach to this class of solutions, see [35].

3.3 Validity of solutions

We are interested in supersymmetric solutions of the four-dimensional effective theory defined by the superpotential (2.11) and the Kähler potential (2.12). In order for these solutions to be at all trustworthy, we need to have a small value for the string coupling g, given through (3.5). Otherwise, nonperturbative string effects may radically change the structure of the low-energy theory. As we will describe in detail in the following section, we have found families of solutions with parametrically small coupling g. To have a completely controllable solution it is also necessary to show that α' corrections are suppressed. This is possible in geometric compactifications when the size of the compactification manifold can be tuned to be arbitrarily large, suppressing α' corrections. Unfortunately, in the models we construct here, it seems very difficult to attain control over α' corrections when nongeometric fluxes are incorporated. A full string construction of these models is probably necessary to address this question.

We can, however, address the issue of control somewhat further from the 4D point of view. The next critical question in effective field theory calculations in a typical flux compactification is whether the moduli are stabilized at masses light enough to render the computation reliable when perturbative string effects are included – that is, whether the effective field theory describes effects at energy scales which are controllably smaller than the energy scales of higher Kaluza-Klein modes, winding modes, and excited string modes. The analogous question for a compactification incorporating nongeometric twists is more subtle as in the absence of a well-defined critical sigma model we do not know precisely how to estimate the mass scales of the string modes (in general the distinction between Kaluza-Klein and other inherently stringy modes such as winding modes will break down). Given some such concrete stringy solution, we might hope in principle to find a rough estimate of the scale of the string modes to compare with the masses of the modes retained in the four-dimensional effective theory.

We have found sets of supersymmetric solutions to the effective field theory which have tunably small string coupling, cosmological constant, and masses for the moduli, as we will detail in the following section. These solutions have some very interesting properties. As the string coupling can be made arbitrarily small, they are well in the perturbative regime. The mass scales computed in the effective theory can also be made parametrically small, although, again, we do not have a reliable method of computing the scale of the effective UV cutoff in our models for comparison. Here we take, as a zeroth-order estimate, the scale of heavier modes to be set by those scales which, in the absence of flux, would set the masses of momentum and winding modes on the torus. The mass scales in our families of solutions can indeed be made parametrically lighter than this estimate for the cutoff. Our solutions do contain non-nilpotent algebras, however, (as do [32, 33]) and thus are in the class of vacua which seem most difficult to lift to ten dimensions. By analogy with the geometric cases discussed in [7], we might be concerned that our naïve method of estimating the cutoff scale, based on an underlying torus geometry, may be inaccurate, so that some of the moduli masses may be comparable to the cutoff scale. In this case, our vacua would need to be augmented with other modes with comparable masses to the fields already included in the effective four-dimensional theory in order to construct a full-blown string model. Furthermore, as mentioned above, we do not have any reason to believe α' corrections are controlled in these vacua. On the other hand, the existence of supersymmetric vacua with parametrically controlled string coupling, moduli masses, and cosmological constant suggests strongly that these vacua can be lifted in some way to string theory.

4. Families of nongeometric vacua

In this section we discuss two particularly interesting infinite families of solutions to the

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equations of motion presented in the previous section. These infinite families precisely saturate the tadpole (which as discussed in section 2 is equal to 16) without the inclusion of additional D-brane sources.

The fluxes which lead to our interesting infinite families of solutions are all modular equivalent to fluxes having

$$P_2(\tau) = b(\tau - 1)^3 \tag{4.1}$$

for some integer b, i.e. all the b_i are equal. This form for the b_i imposes restrictive constraints on the c_i. If in addition $c_3 = 0$, there are only two possible choices for the remaining c_i which satisfy the constraints (2.16)–(2.22). These two options are, in the notation $(c_0, (c_1, \tilde{c}_1), (c_2, \tilde{c}_2), c_3)$,

$$\text{either} \quad (2(n+m), (-2m, 2n), (0, 2m), 0) \tag{4.2}$$

$$\text{or} \quad (2(n+m), (0, 2m), (2n, 2n), 0). \tag{4.3}$$

Each of these choices for the c_i leads to an interesting set of physical solutions, as we will see. Given these choices for the b_i and the c_i, one must then choose a_i which satisfy the RR constraints (2.14)–(2.15). A particularly simple way to solve the RR constraints is to take $a_0 = 16/b$, with the remaining $a_i = 0$. Since a_0 and b must be (even) integers, the only possible choices for b are 2, 4, and 8. There are other choices one may make for the a_i; we merely choose this one for simplicity and because we can exactly solve for the moduli.

The first family of solutions we consider is (4.2), with the a_i chosen as in the previous paragraph. Using the notation (a_0, a_1, a_2, a_3), (b_0, b_1, b_2, b_3), $(c_0, (c_1, \tilde{c}_1), (c_2, \tilde{c}_2), c_3)$, the fluxes here are

$$(16/b, 0, 0, 0),\ (b, b, b, b),\ (2(n+m), (-2m, 2n), (0, 2m), 0). \tag{4.4}$$

These fluxes yield physical solutions for the moduli

$$\tau_0 = \left(\frac{n+m}{m}\right) \pm \left(\frac{n}{m}\right) i, \tag{4.5}$$

$$S_0 = \frac{m^3}{b^2 n^3} \left(-8 \pm 2i\right), \tag{4.6}$$

$$U_0 = \frac{m}{bn^2}(4 \pm 2i). \tag{4.7}$$

The $\pm$ in the imaginary parts should be picked to make the imaginary parts positive, i.e. $+$ when $m/n > 0$ and $-$ when $m/n < 0$. The same sign must be picked in all three moduli to ensure a solution to the equations of motion. This family of solutions has string coupling

$$g = \left|\frac{b^2 n^3}{2m^3}\right| \tag{4.8}$$

and cosmological constant

$$\Lambda = -\left|\frac{3b^3 n^6}{16m^3}\right|. \tag{4.9}$$

This family is thus easy to tune to small string coupling and cosmological constant by fixing n and taking m to be large. Notice that it is impossible to tune both g and Λ to

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be finite; there must be an accumulation point for one or both quantities at either zero or infinity in any infinite set.

Meanwhile, the fields τ, S, U have a mass matrix given by

$$M_{ij} = \begin{pmatrix} \pm\frac{21b^3n^4}{16m} & 0 & 0 & \frac{3b^5n^8}{32m^5} & 0 & \frac{-3b^4n^7}{32m^3} \\ 0 & \pm\frac{33b^3n^4}{16m} & \frac{-3b^5n^8}{16m^5} & 0 & 0 & 0 \\ 0 & \frac{-3b^5n^8}{16m^5} & \pm\frac{b^7n^{12}}{32m^9} & 0 & \pm\frac{3b^6n^{11}}{64m^7} & 0 \\ \frac{3b^5n^8}{32m^5} & 0 & 0 & \pm\frac{b^7n^{12}}{32m^9} & 0 & \pm\frac{3b^6n^{11}}{32m^7} \\ 0 & 0 & \pm\frac{3b^6n^{11}}{64m^7} & 0 & \pm\frac{3b^5n^{10}}{32m^5} & 0 \\ \frac{-3b^4n^7}{32m^3} & 0 & 0 & \pm\frac{3b^6n^{11}}{32m^7} & 0 & \pm\frac{3b^5n^{10}}{16m^5} \end{pmatrix}, \tag{4.10}$$

in the basis $\{\mathrm{Re}\,\tau, \mathrm{Im}\,\tau, \mathrm{Re}\,S, \mathrm{Im}\,S, \mathrm{Re}\,U, \mathrm{Im}\,U\}$. The choices of sign $\pm$ in the matrix elements are determined by the choices of sign in the moduli, (4.5)–(4.7). The eigenvalues of M_{ij} come in pairs, scaling as m^{-1}, m^{-5}, and m^{-9} in the controllable limit where $m \to \infty$ with n fixed. Thus all the fields in this family of solutions have parametrically light masses. Our zeroth-order estimate for the cutoff scale, by contrast, scales as m^0.

The second two-parameter family, with the c_i as given in (4.3), is similar to the one above. The fluxes are

$$(16/b, 0, 0, 0), \ (b, b, b, b), \ (2(n+m), (0, 2n), (2m, 2m), 0), \tag{4.11}$$

and it is easy to check that the moduli

$$\tau_0 = -\left(\frac{n+m}{m}\right) \pm \left(\frac{n+2m}{m}\right) i, \tag{4.12}$$

$$S_0 = \frac{1}{b^2}\left(\frac{m}{2m+n}\right)^3 (8 \pm 2i)\,, \tag{4.13}$$

$$U_0 = \frac{m}{b(2m+n)^2}(-4 \pm 2i) \tag{4.14}$$

solve the F-flat equations. As above, the $\pm$ in the imaginary parts of these moduli should be picked to keep the imaginary parts positive; the plus sign should be used when $m > 0$, $n > -2m$ and the minus sign should be used when $m < 0$, $n > -2m$. Again, the same sign must be picked in all three moduli to give a solution to the equations of motion. These values for the moduli give string coupling

$$g = \left|\frac{b^2(2m+n)^3}{2m^3}\right| \tag{4.15}$$

and cosmological constant

$$\Lambda = -\left|\frac{3b^3(2m+n)^6}{16m^3}\right|. \tag{4.16}$$

The masses for the moduli in these vacua are again parametrically light. These two families have identical spectra but are not equivalent under modular transformations.

There are more general choices one can make for the a_i than in the two families discussed explicitly above. For example, any modular transformation $S \to S + q$ or $U \to U + q$ will take the fluxes of (4.4), (4.11) to flux sets with more a_i nonzero.

It is particularly interesting to consider the effect of fractional shifts of the form $U \to U + (1/q)$, on these families. These shifts are permissible whenever c_i/q is an even integer for all c_i. Every time we can perform such a shift, we get a physically inequivalent theory with identical spectrum. Each fractional shift yields another infinite family of vacua, with different values for the flux parameters a_i and for the expectation value of $\mathrm{Re}\, U$. Clearly, the number of such shifts we can perform is controlled by the factors of the c_i fluxes. But because the c_i appear only homogenously in the constraint equations, we are free to scale them up to arbitrarily large values. As we scale up m and n, we will encounter numbers that have an increasingly large number of factors, and be able to perform more such shifts. Thus, as m and n increase, the degeneracy of these vacua increases, becoming infinite in the $m, n \to \infty$ limit. Degeneracies due to fractional shifts of the axions are in fact a general, though little-remarked, feature of flux compactifications; the novelty in the present case is that the tadpole cancellation conditions fail to prevent arbitrarily large degeneracies.

In this section we have derived infinite families of solutions to the four-dimensional field theory associated with general fluxes. As we have discussed, it is not clear that we can take even those solutions with small g at face value as solutions of a complete string theory compactification. It is furthermore entirely possible that even if these solutions are valid for small fluxes, they may break down in the limit of large NS-NS flux as back reaction effects become large. Although the average energy density in the internal space due to the fluxes, given by the cosmological constant, can remain small, as the fluxes become infinite one might expect that there are regions where the local energy density becomes large enough to necessitate an accurate treatment of back reaction.

5. Statistics of generic vacua

So far we have described some special classes of solutions to the equations of motion (3.1), (3.2), (3.3). To investigate the properties of general solutions, we proceed numerically. Numerical computation of solutions for flux vacuum equations of this type requires three ingredients. First, we must generate a set of (even) fluxes satisfying the constraint equations (2.14), (2.22). Second, we must numerically solve the equations (3.1), (3.2), (3.3) for the moduli τ, S, U. Finally, we must impose a gauge-fixing of the modular symmetries (A.1)–(A.6) so that we do not count the same "vacuum" twice.

In previous analogous work on the statistics of IIB flux compactifications [28, 36, 37] only a finite number of solutions were compatible with a fixed tadpole bound L. Thus, L acted as a natural cutoff, and it was natural to investigate the growth of the number of solutions as a function of L. In the case we are investigating here, as we have seen in the previous section, there are infinite families of solutions even at fixed tadpole $L = 16$. To proceed numerically we must impose an artificial cutoff on the set of allowed fluxes. In order to extract physically meaningful results, we then need to consider only those quantities which approach well-defined limits as the cutoff is taken to infinity.

Generating fluxes satisfying the constraints is straightforward. Fixing an upper bound N for the fluxes, so that

$$|a_i|, |b_i|, |c_i| \leq N \,, \tag{5.1}$$

we can scan through an independent subset of the fluxes and solve for the remaining dependent fluxes using some of the constraints in time polynomial in N. In general, any particular algebraic solution to the constraint equations is only valid if certain quantities are nonvanishing. For example, solving (2.14) for a_0 is only possible if $b_3 \neq 0$. We have generated fluxes using several different subsets of the equations, and have confirmed that the fluxes missed in this process comprise an increasingly small fraction of the set of allowed fluxes as N is increased.

Numerically solving the equations (3.1), (3.3) is also straightforward in principle. Again, however, any specific approach to solving the equations assumes certain quantities to be nonvanishing. We have carried out a systematic search for solutions to the F-flat equations which stabilize all moduli at tree level for fluxes within the region (5.1). The stabilization of all moduli is generic, although as we have mentioned in section 3.2 there are some cases of special interest which do not fall into this category.

As discussed in, for example, [28, 37], there are two ways of imposing gauge-fixing. On the one hand, we can generate all possible fluxes satisfying the constraints (up to our cutoff), numerically solve the vacuum equations for all fluxes, and then select only those solutions which live in a fixed modular region in τ, S, U space. On the other hand, we can perform the gauge-fixing at the level of the fluxes, choosing only fluxes which satisfy a particular gauge-fixing condition. In general, if we can gauge-fix at the level of the fluxes, such that we can efficiently generate only fluxes in the given modular region, our search will be much more efficient, since otherwise most of our computer time is spent scanning regions outside the modular domain; this problem becomes worse as the scale of the fluxes increases and more copies of the modular domain are probed.

The challenge in gauge-fixing at the level of the fluxes is that there is not always a simple gauge-fixing choice for the fluxes. In the problem we are interested in here, it is easy to fix the modular transformations $S \to S + n$, $U \to U + m$ by imposing conditions on the fluxes. It is less straightforward, however, to fix the $\mathrm{SL}(2, \mathbb{Z})$ symmetry on τ at the level of the fluxes. We have used the following approach to gauge-fix this symmetry: when $P_2(\tau)$ has a pair of complex roots, we can use it to determine a reference value τ_r which is the root of P_2 in the upper-half complex plane. We then gauge-fix the $\mathrm{SL}(2, \mathbb{Z})$ symmetry by requiring τ_r to be in the standard fundamental domain $\mathcal{F} = \{\tau : |\tau| \geq 1, -1/2 < \mathrm{Re}\,\tau \leq 1/2, |\tau| = 1 \Rightarrow \mathrm{Re}\,\tau \geq 0\}$. When $P_2(\tau)$ has all real roots, but $P_3(\tau)$ has a pair of complex roots, we choose τ_r to be the complex root of P_3 with positive imaginary part. We then again fix the $\mathrm{SL}(2, \mathbb{Z})$ symmetry by taking $\tau_r \in \mathcal{F}$. The complexity of the roots of a cubic polynomial depends on a discriminant inequality on the coefficients, much as for a quadratic polynomial. Thus, the subset of fluxes which cannot be gauge-fixed in this way represents a fraction of order 1 of the full set of fluxes.

We have generated sets of fluxes up to $N = 30$ which satisfy the following explicit modular and sign fixing conditions

- $\tau_r \in \mathcal{F}$
- $c_3 \geq 0$, if $c_3 = 0$ then $b_3 \geq 0$

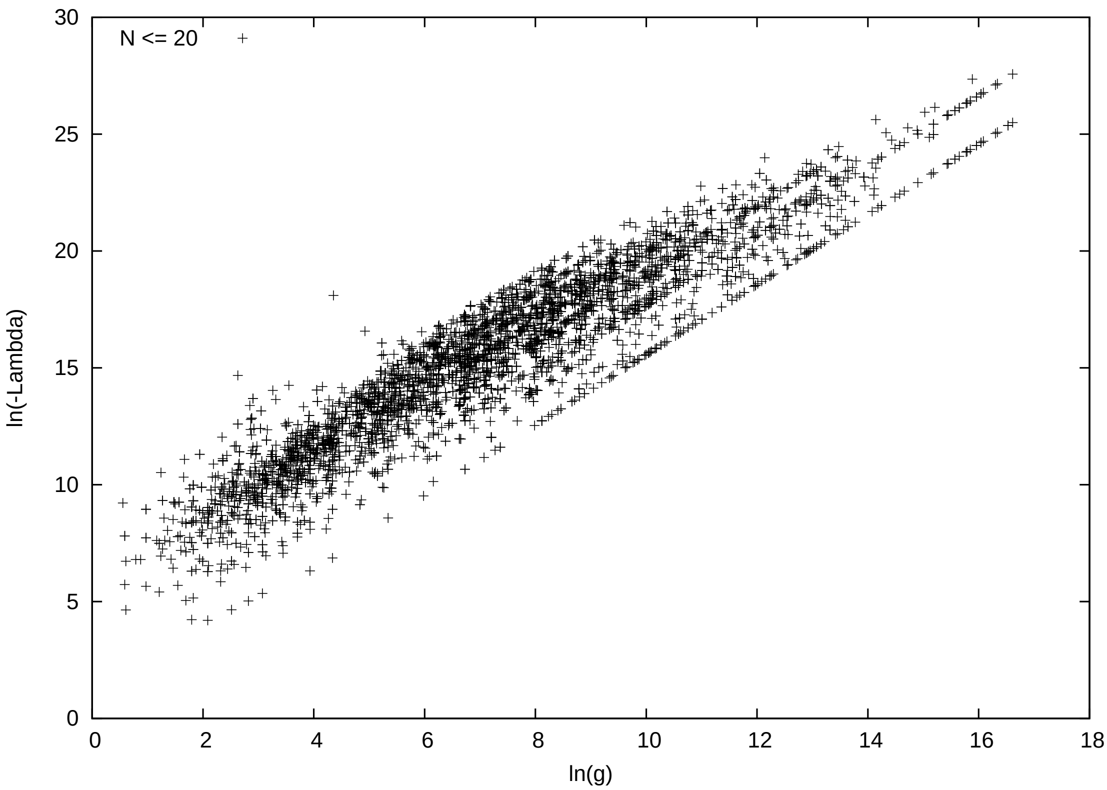

Figure 1: Log-log plot of g, Λ for gauge-fixed fluxes up to $N = 20$

- $b_0 \geq 0$, if $b_0 = 0$ then $c_0 \geq 0$
- $(a_0, a_3) = \alpha(b_0, b_3) + 3\beta(c_0, c_3)$, where $0 \leq \alpha, \beta < 1$

Restricting to these fluxes completely fixes the modular freedom, though again some sets of measure zero are lost. Here we are fixing two choices of sign, both the choice of overall sign for the fluxes, which relates two different descriptions of the same vacuum, as well as the global transformation (A.7), which relates two inequivalent but degenerate vacua. Therefore for each solution we find, there is an additional solution related by the transformation (A.7). We find that the number of gauge-fixed fluxes in the region (5.1) scales roughly as N^4.

We have explicitly solved the equations of motion (3.1), (3.3) for this set of gauge-fixed fluxes. We find that roughly 20% of all fluxes yield physical supersymmetric solutions. This fraction did not change significantly as N increased. A plot of the distribution of string couplings g and cosmological constants Λ is shown in figure 1, for the data up to $N = 20$. As can be seen from the figure, large string coupling is correlated with large Λ. This is not surprising, since $g = 1/\text{Im } S_0$ appears as a factor of e^K. In addition one may see from studying the equations of motion that g does not scale with P_3, while Λ scales as P_3^3, so that rescaling the c_i increases the cosmological constant and correspondingly decreases U

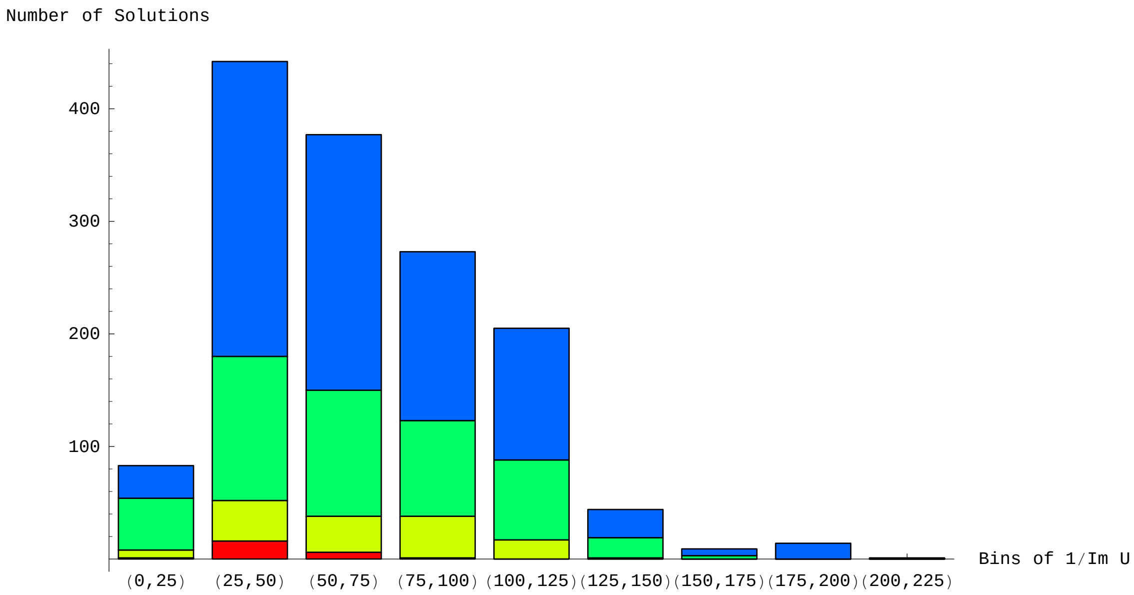

Figure 2: Distribution of values of $1/\mathrm{Im}\, U$ as a function of maximum flux N for solutions within a particular range of g and Λ. Successive shaded regions correspond to $N = 8, 12, 16, 20$ respectively.

without affecting g. This demonstrates that the distribution shown in figure 1 will have a tail reaching up to $-\Lambda \to \infty$ at fixed g as $N \to \infty$.

What is perhaps even more interesting is that the overall shape of the distribution shown in figure 1 does not change appreciably as N increases. As far as we can tell from our sample, the distribution of vacua in g-Λ space is independent of the scale N of the fluxes. This implies in particular that unless there is some dramatic qualitative change at much larger N, there are an infinite number of solutions of the F-flat equations in fixed finite regions of g-Λ parameter space. Moreover, the vacuum expectation values for the non-compact scalar fields $\mathrm{Im}\, \tau$ and $\mathrm{Im}\, U$ also accumulate according to a distribution whose shape is apparently independent of N. This leads to an infinite accumulation of solutions in fixed finite regions of parameter space. In figures 2 and 3 we select one bounded region in g-Λ space and show how the number of solutions within given ranges of $\mathrm{Im}\, \tau$ and $\mathrm{Im}\, U$ increases with the maximum flux N. The region in g-Λ space we have chosen to illustrate in the figures is $10 < \ln(-\Lambda) < 15$, $1 < \ln g < 8$. In figure 2 we show the distribution of $\mathrm{Im}\, U$ for solutions which lie within this bounded region in g-Λ space. In figure 3, we show the distribution of $\mathrm{Im}\, \tau$ for the same set of solutions. The data have been binned into ranges of useful size for both variables. The gradations of color/grayscale in the graphs show how the solutions accumulate as a function of maximum flux size N; we have plotted the data for $N = 8, 12, 16$, and 20, with a separate color/shade for each successive value of N. As is evident in both figures, the distribution of vacua appears to be fairly independent

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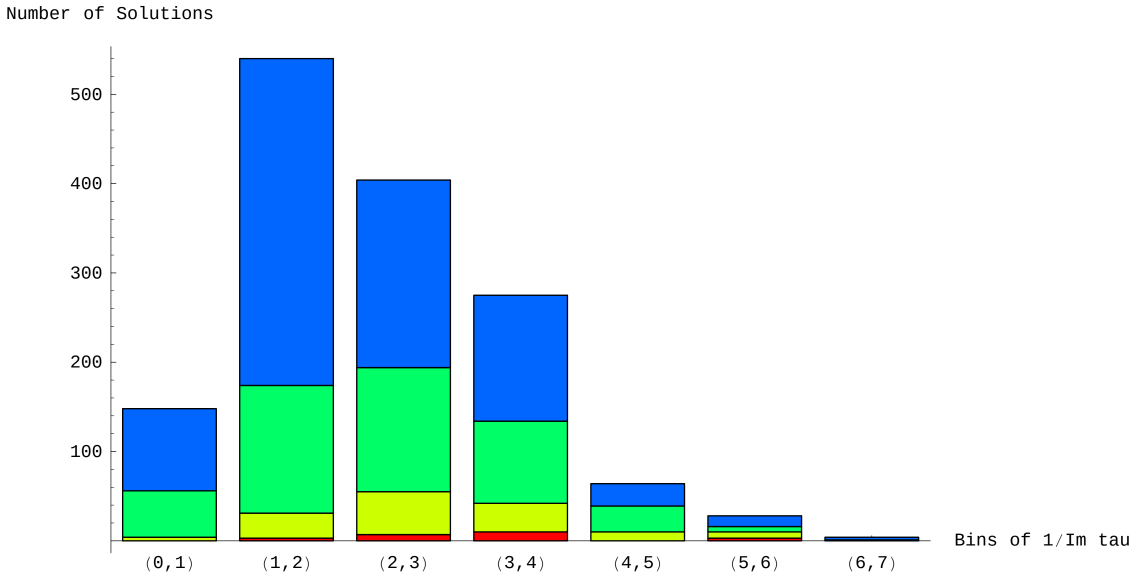

Figure 3: Distribution of values of $1/\mathrm{Im}\,\tau$ as a function of maximum flux N for solutions within a particular range of g and Λ. Successive shaded regions correspond to $N = 8, 12, 16, 20$ respectively.

of the scale N of the fluxes, and exhibits no strong indication of running to the boundaries of moduli space. While these "vacua" are completely untrustworthy, lying at large string coupling, this illustrates that for this model there seem to be an infinite number of distinct tree-level SUSY solutions for a family of gauged supergravity theories in a finite volume of moduli space. This is the only example we know of where even in the tree-level low-energy approximation there is such an infinite family of vacua. If these vacua correspond to good vacua for a full nonperturbative string theory, it would seem to contradict the expectation of [23] that there are only a finite number of vacua compatible with any finite region of physical parameter space. It would be interesting to study these solutions further to ascertain whether they have a specific physical nonperturbative instability. Another interesting feature of the distribution of solutions shown in figure 1 is that it does not contain any solutions with small g, Λ. This stands in contrast to the cases where both $P_2(\tau)$ and $P_3(\tau)$ have three real roots, as we now discuss.

Although the above gauge-fixing procedure breaks down in the case where all roots of $P_2(\tau)$ and $P_3(\tau)$ are real, we have generated a representative sample of fluxes in this category up to $N = 20$ and numerically solved the equations of motion. It is then possible to remove the modular redundancy by choosing the solution for τ to lie within $\mathcal{F}$, and then imposing the other gauge-fixing conditions. The resulting distribution of solutions is shown in figure 4. This distribution has the same broad features as that shown in figure 1. In particular, we still see evidence for an infinite number of distinct solutions in finite regions

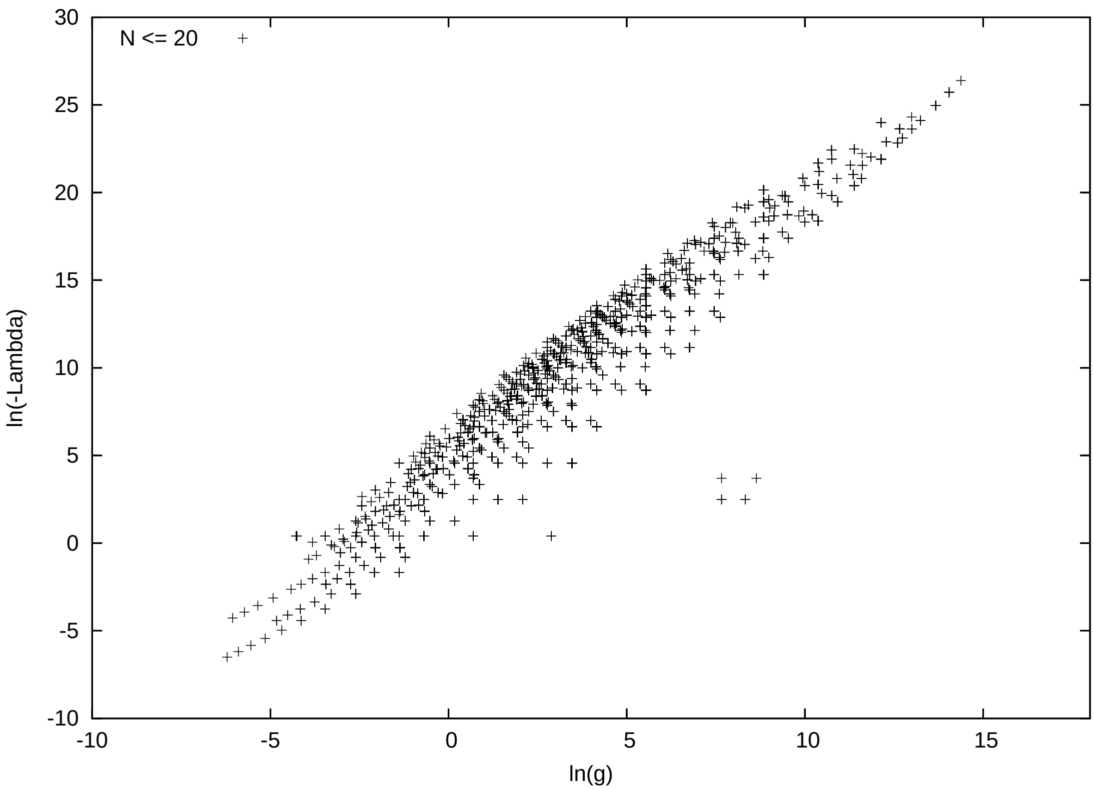

Figure 4: Log-log plot of g, Λ for fluxes where P_2, P_3 have real roots up to $N = 20$

of g-Λ-space. The only major difference between this case and the complex root case in figure 1 is that here, much smaller values of g and Λ appear. Indeed, many of the points in figure 4 are members of the controllable families discussed in the previous section. It would be nice to have a clearer quantitative understanding of why small values of g and Λ seem so much more difficult to realize in the case where $P_2(\tau)$, $P_3(\tau)$ have complex roots.

At this point it is perhaps worthwhile to reiterate that the results in this section, for the fraction of supersymmetry-preserving flux sets and the relative number of supersymmetric solutions in different regions of parameter space, are independent of N for large enough N, and are therefore apparently well-defined as the cutoff on the maximum flux is removed.

6. Conclusions

The main goal of this paper has been to investigate supersymmetric vacua of the $\mathcal{N} = 1$ effective theory developed in [9]. We find that generic supersymmetric solutions have the following properties: all moduli are stabilized at tree level; the solutions are not gauge-equivalent to geometric compactifications in any duality frame; the string coupling and four-dimensional cosmological constant are not particularly small; and there appear to be an infinite number of solutions in finite regions of parameter space without vanishing or infinite limits. In addition, we have found infinite families of vacua which have parametrically controllable string coupling and mass scales, though we do not have any reason to believe that the α' expansion is under control for these vacua..

The apparently infinite number of solutions in bounded regions of parameter space is perhaps a bit surprising, and seems to conflict with the intuition underlying recent speculations regarding finiteness of the landscape of string vacua [23, 38]. Indeed, if there are an infinite number of valid string vacua in a finite range of parameter space in effective field theory, with sufficiently small (or negative) cosmological constants, it is difficult to avoid an infinite transition rate into those vacua via vacuum decay processes (though not impossible, if for example increasing a parameter such as a flux sufficiently quickly increases the height or width of the potential barrier separating these vacua from the physical vacuum). It is therefore worthwhile to discuss a number of possible reasons why the solutions we have found here may not represent true supersymmetric string vacua.

To begin with, as we have discussed, it is quite possible that the four-dimensional effective field theories studied here simply cannot be lifted to a full string background, even with the inclusion of other modes with masses comparable to those of the moduli. Even if these field theories can be lifted to geometric or nongeometric backgrounds, given the difficulty of achieving modular invariance in nongeometric asymmetric orbifold constructions [20, 39] it is conceivable that the class of generalized flux compactifications we consider here must obey unexpected constraints at the one-loop level which would eliminate many or all of the tree-level solutions given here. It is also possible that we have missed a tree-level constraint on the allowed fluxes, although it is somewhat difficult to see how further constraints might arise. Actually constructing string backgrounds which incorporate nongeometric fluxes is probably the most promising way of analyzing this set of possibilities further, and we leave this as an open problem.

Another possibility is that we have misidentified the modular redundancy of the theory, and that many of the distinct solutions which we find should in fact be considered to be equivalent descriptions of the same solution. We can conceptually separate two different ways in which the number of vacua becomes large: first, the fractional axionic shifts which give potentially large numbers of degenerate vacua with identical g and Λ, and second, the increasing density of distinct values of g and Λ within a finite region as the maximum flux N is increased. There are no known symmetries which would identify these vacua, and our numerical work shows no evidence of extra redundancies associated with unknown modular equivalences which would reduce the number of inequivalent vacua to a finite number. It is conceivable, however, that such a symmetry exists, which would reduce the number of physically inequivalent solutions.

Yet another possibility is the destabilization of the tree-level solutions by the inclusion of either α' effects or neglected momentum or winding modes. If our solutions do lift to full string backgrounds, we expect in general that α' effects will be significant. In conventional flux compactifications, one can often work in a large volume limit, where α' corrections are highly suppressed; we are unable to do that here. Indeed, in simple examples nongeometric fluxes typically stabilize length scales of the compactification manifold at the string scale by relating radii to their duals under monodromy, so we expect α' corrections to generically be important. Finally, it is possible that we should not allow the NS-NS fluxes to become arbitrarily large, as such a flux configuration might lead to large local backreaction, as we have discussed briefly at the end of section 4.

Despite all these possible problems in extending the solutions we have found to full-fledged string theory solutions, the generalized fluxes appearing in the superpotential seem to capture essentially topological features of the string compactification, and it seems to us likely that the vacua found here do capture the main features of a new class of string compactifications. Even if these four-dimensional effective theories do not include all the physics of the full string theory, they may point the way to a new and interesting class of constructions of string vacua. Certainly, the question of which four-dimensional effective supergravity theories can be lifted to complete string theories is a very important one which deserves more attention at this time. Whether this question is best resolved by a top-down (landscape [40]) approach of attempting to characterize the most general structure of string theory compactifications, or by a bottom-up (swampland [24, 41]) approach of understanding which four-dimensional theories have a good UV completion remains to be seen. In any case, a full characterization of the space of string vacua which might be compared productively with experiment requires a resolution of this question. It is our hope that the solutions and caveats which we have developed here may provide a useful set of test cases to better understand the boundaries of the landscape.

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A. Modular transformations

Here we give the transformation properties of the fluxes under the modular transformations described in section 3.

First, consider the shift of the axiodilaton $S \to S + n$. This shift of the modulus is accompanied by a shift of the flux parameters,

$$a_i \to a_i + nb_i, \tag{A.1}$$

with b_i, c_i invariant, leaving the superpotential invariant under the shift.

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Second, consider the shift $U \to U + n$. Under this transformation the fluxes transform as

$$
\begin{aligned}
a_0 &\to a_0 - 3nc_0 \\
a_1 &\to a_1 + n(2c_1 - \tilde{c}_1) \\
a_2 &\to a_2 + n(2c_2 - \tilde{c}_2) \\
a_3 &\to a_3 - 3nc_3,
\end{aligned}
\tag{A.2}
$$

with b_i, c_i invariant.

Finally, consider the shift $\tau \to \tau + n$, which is a geometric transformation in type IIB. The transformation laws for the a_i, b_i are, as usual,

$$
\begin{aligned}
a_0 &\to a_0 + 3a_1 n + 3a_2 n^2 + a_3 n^3 \\
a_1 &\to a_1 + 2na_2 + n^2 a_3 \\
a_2 &\to a_2 + na_3 \\
a_3 &\to a_3,
\end{aligned}
\tag{A.3}
$$

while the transformations of the c_i are slightly more involved:

$$
\begin{aligned}
c_0 &\to c_0 - 2nc_1 + n\tilde{c}_1 - 2n^2 c_2 + n^2 \tilde{c}_2 + n^3 c_3 \\
c_1 &\to c_1 - n\tilde{c}_2 + nc_2 - n^2 c_3 \\
\tilde{c}_1 &\to \tilde{c}_1 - 2nc_2 + n^2 c_3 \\
c_2 &\to c_2 - nc_3 \\
\tilde{c}_2 &\to \tilde{c}_2 + nc_3 \\
c_3 &\to c_3.
\end{aligned}
\tag{A.4}
$$

The separate transformations of c_1 and $\tilde{c}_1$ can be deduced most easily from IIB, where the shift $\tau \to \tau + n$ is a large diffeomorphism and the c_i are all given by Q_c^{ab} on different cycles. By taking Q to transform as a mixed tensor under diffeomorphisms one may obtain the transformation rules above for the integrated fluxes.

There is also an inversion symmetry $\tau \to -1/\tau$, under which

$$
\begin{aligned}
a_0 &\to a_3 \\
a_1 &\to -a_2 \\
a_2 &\to a_1 \\
a_3 &\to -a_0
\end{aligned}
\tag{A.5}
$$

and similarly for the b_i; meanwhile,

$$
\begin{aligned}
c_0 &\to c_3 \\
c_1 &\to -c_2 \\
c_2 &\to c_1 \\
c_3 &\to -c_0.
\end{aligned}
\tag{A.6}
$$

Under this transformation $W \to -(1/\tau^3)W$, which combined with the change in the Kähler potential (2.12) leaves the scalar potential invariant.

The last factor in the modular group, as described in section 3, is a $\mathbb{Z}_2$ transformation which flips the signs of all fluxes, leaving the moduli invariant.

One may check that all the constraints are preserved under the above modular transformations.

In section 5 we fix in addition the global transformation which takes $\tau, S, U \to -\bar{\tau}, -\bar{S}, -\bar{U}$. Under this transformation the fluxes which multiply odd powers of the moduli in the superpotential switch signs,

$$a_1, a_3, b_0, b_2, c_0, c_2, \tilde{c}_2 \to -a_1, -a_3, -b_0, -b_2, -c_0, -c_2, -\tilde{c}_2 \tag{A.7}$$

so that $W \to \bar{W}$.

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